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LETTER TO THE EDITORS

COMMENTS ON THE PAPER "THEORETICAL STUDY OF LAMINAR FILM CONDENSATION OF FLOWING VAPOUR"

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I WAS very interested to read the recent paper by I. G. Shekriladze and V. I. Gomelauri [1]. In a paper by J. D. Griffiths, J. W. Phillips and myself [2] we presented an analysis starting from similar assumptions, but in the general solution we included the shear stress arising from C_f^*. We did this because in our own experiments the vapour boundary layer was already fully turbulent at the leading edge of the condensing surface, and therefore we did not expect the effect of C_f^* to be negligible. Our analysis was more limited, however, in that we ignored the value U_s in the momentum stress $j(U_\infty - U_s)$ at an early stage of the solution, and only considered the case of t_w = constant.

The analysis yielded an equation for the film thickness δ at x

$$(Sh)\left(\frac{\delta}{x}\right)^4 + (Dr)\left(\frac{\delta}{x}\right)^3 + (Re_v)\left(\frac{\delta}{x}\right)^2 = 1 \qquad (1)$$

$Sh = \rho^2 g r x^3/4\mu\lambda\,\Delta t$ accounts for the effect of gravity; $Dr = \rho\tau r x^2/\mu\lambda\,\Delta t$ accounts for the effect of friction shear stress $\tau = C_f^*\rho_v U_\infty^2/2$ which is additional to the momentum shear stress; $Re_v = \rho U_\infty x/\mu$ is a two-phase Reynolds number which accounts for the momentum stress $j(U_\infty - U_s) \simeq jU_\infty$ (two-phase because it contains the properties μ and ρ of the liquid but the velocity U_∞ of the vapour). Equation (1) can be solved for δ, and the local Nusselt number found from $Nu_x = x/\delta$. It is also easy to show that the average Nusselt number Nu over a length $x = \mathscr{L}$ is given by

$$Nu = \tfrac{4}{3}(Sh)\psi^3 + \tfrac{1}{2}(Dr)\psi^2 + \tfrac{1}{2}(Re_v)\psi \qquad (2)$$

where $\psi = \delta_{\mathscr{L}}/\mathscr{L}$. Table 1 summarizes the results of film thickness $\delta_{\mathscr{L}}$ at $x = \mathscr{L}$, and of average Nusselt number $Nu = \alpha\mathscr{L}/\lambda$, when some of the external forces acting on the film are neglected.

Equations (11) and (12) correspond to Nusselt's original simple theory which allowed only for gravity. Equations (5) and (6) are similar to Nusselt's solution in which he thought

Table 1

External forces acting on film	$\psi = \delta_{\mathscr{L}}/\mathscr{L}$		Nu	
'General case' Gravity + Friction + Momentum	from equation (1)		from equation (2)	
Gravity + Momentum	$\left[\left\{\left(\frac{Re_v}{8Sh}\right)^2 + \frac{1}{Sh}\right\}^{\frac{1}{2}} - \frac{Re_v}{8Sh}\right]^{\frac{1}{2}}$	(3)	$\left[\left\{\left(\frac{Re_v}{8Sh}\right)^2 + \frac{1}{Sh}\right\}^{\frac{1}{2}} - \frac{Re_v}{8Sh}\right]^{\frac{1}{2}} \times \left[\frac{4}{3}(Sh)\left\{\left(\frac{Re_v}{8Sh}\right)^2 + \frac{1}{Sh}\right\}^{\frac{1}{2}} + \frac{Re_v}{3}\right]$	(4)
Gravity + Friction	from $(Sh)\psi^4 + \frac{1}{3}(Dr)\psi^3 = 1$	(5)	$\frac{4}{3}(Sh)\psi^3 + \frac{1}{2}(Dr)\psi^2$	(6)
Momentum	$\left(\frac{4}{Re_v}\right)^{\frac{1}{2}}$	(7)	$(Re_v)^{\frac{1}{2}}$	(8)
Friction	$\left(\frac{3}{Dr}\right)^{\frac{1}{3}}$	(9)	$\frac{1}{2}(9Dr)^{\frac{1}{3}}$	(10)
Gravity	$\left(\frac{1}{Sh}\right)^{\frac{1}{4}}$	(11)	$\frac{4}{3}(Sh)^{\frac{1}{4}}$	(12)

that he had allowed for all the vapour drag, not realizing that the momentum drag term was mainly responsible for the shear stress on the film.

Our equation (8) corresponds to equation (9) of Shekriladze and Gomelauri and the latter can be put in the form

$$Nu = \left(\frac{Re_v}{1 + N}\right)^{\frac{1}{2}}. \tag{13}$$

Since $N \ll 1$ except for liquid metals, there is apparently good but nor exact agreement between the two solutions.

Our equation (4) corresponds to equation (20) of Shekriladze and Gomelauri and the latter can be put in the form

$$Nu = (Re_v)^{\frac{1}{2}} \frac{\sqrt{2}\left[2 + \left(1 + 64\frac{Sh}{Re_v^2}\right)^{\frac{1}{2}}\right]}{3\left[1 + \left(1 + 64\frac{Sh}{Re_v^2}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}} \tag{14}$$

There is no obvious way of showing whether this equation and equation (4) in Table 1 are identical or not, but sample calculations for typical particular situations give similar numerical results.

It is interesting to note that equation (14) for zero gravity, i.e. for a horizontal plate for which $Sh = 0$, reduces to our equation (8) and not to equation (13). This suggests some inconsistency in their solution. It is also surprising that equations (13) and (14) do not contain U_s. The authors say that in deriving these equations they have used their equation (4), and not (3), but it is difficult to see at what point they have eliminated U_s. It is a pity that their derivations and approximations, if any, are not even in outline.

REFERENCES

1. I. G. SHEKRILADZE and V. I. GOMELAURI, Theoretical study of laminar film condensation of flowing vapour, *Int. J. Heat Mass Transfer* **9**, 581–591 (1966).
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Y. R. MAYHEW

Mechanical Engineering Department
University of Bristol